Date: 24-04-2018   Dept. No.   Max. : 100 Marks
Time: 01:00-04:00

PART-A

Answer ALL the questions   **(10 x 2=20)**

1. Evaluate $\int \sec^6 x \, dx$.

2. Evaluate $\int_0^1 x \left(1 - x^2\right)^{\frac{1}{2}} dx$.

3. Evaluate $\int_1^2 \int_1^x xy^2 \, dy \, dx$.

4. If $u = x^2 - y^2$ and $v = x^2 + y^2$, then find $\dfrac{\partial(x, y)}{\partial(u, v)}$.

5. Prove that $\beta(m,n) = \beta(n,m)$.

6. Prove that $\Gamma(n+1) = n!$.

7. Test for convergency the series $\displaystyle\sum_{n=0}^{\infty} \dfrac{n^3 + 1}{2^n + 1}$.

8. State Raabe's Test.

9. Expand $(1-x)^{-3}$.

10. Expand $\dfrac{e^x + e^{-x}}{2}$.

PART-B

Answer any FIVE questions   **(5 x 8=40)**

11. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

12. Find the area of the cardioid $r = a(1 + \cos\theta)$.

13. Evaluate $\iint_R (x - y)^4 e^{x+y} \, dx \, dy$ where R is the square with vertices $(1,0),(2,1),(1,2)$ and $(0,1)$.

14. Express $\displaystyle\int_0^1 x^m \left(1 - x^n\right)^p dx$ in terms of gamma function and evaluate $\displaystyle\int_0^1 x^{\frac{3}{2}} (1-x)^{\frac{5}{2}} dx$.

15. Test for the convergency or divergency the series $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n^4 + 1} - \sqrt{n^4 - 1}\right)$.

16. Examine the convergence of $\dfrac{1^2}{2^2} + \dfrac{1^2}{2^2} \cdot \dfrac{3^2}{4^2} + \dfrac{1^2}{2^2} \cdot \dfrac{3^2}{4^2} \cdot \dfrac{5^2}{6^2} + \dots$.

17. Sum the series $\dfrac{1^2}{1!} + \dfrac{1^2 + 2^2}{2!} + \dfrac{1^2 + 2^2 + 3^2}{3!} + ... + \dfrac{1^2 + 2^2 + 3^2...n^2}{n!} + ...$ to ∞.

18. Show that $\dfrac{5}{1.2.3} + \dfrac{7}{3.4.5} + \dfrac{9}{5.6.7} + ...\infty = 3\log 2 - 1$.

PART-C

Answer any TWO questions (2 x 20=40)

19. (a) Derive the reduction formula for $\int \cos^n x \, dx.$ **(10)**

    (b) Evaluate $\displaystyle\int_0^{\pi/2} \log \sin x \, dx$. **(10)**

20. (a) Evaluate $\displaystyle\iint xy \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ by transforming into polar coordinates. **(10)**

    (b) By changing the order of integration, evaluate $\displaystyle\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y} \, dx \, dy.$ **(10)**

21. (a) State and prove the relation between gamma and beta functions. **(15)**

    (b) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^7 \theta \cos^7 \theta \, d\theta$ by using gamma function. **(5)**

22. (a) Discuss the convergence of the series $\dfrac{1}{1+x} + \dfrac{1}{1+2x^2} + \dfrac{1}{1+3x^3} + ...$ for positive values of x. **(10)**

    (b) Find the sum to infinity of the series $1 + \dfrac{3}{4} + \dfrac{3.5}{4.8} + \dfrac{3.5.7}{4.8.12} + ...\infty.$ **(10)**

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